

# GEOPHYSICAL COORDINATE TRANSFORMATIONS

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## Abstract .

The use of a vector-matrix formalism to describe the transformation from one cartesian coordinate system to another results in simple-to-use and easy-to-understand relationships. Furthermore, the required matrix transformations may be derived directly. The common coordinate systems in use in solar-terrestrial relationships are described and then the transformation matrices required to convert vectors in one system to another are derived.

## Introduction

Many different coordinate systems are used in experimental and theoretical work on solar- terrestrial relationships. These coordinate systems are used to display satellite trajectories, boundary locations, and vector field measurements. The need for more than one coordinate system arises from the fact that often various physical processes are more understood, experimental

data more ordered, or calculations more easily performed in one or another of the various systems. Frequently, it is necessary to transform from one to another of these systems. It is possible to derive the transformation from one coordinate system to another in terms of trigonometric relations between angles measured in each system by means of the formulas of spherical trigonometry (*Smart*, 1944). However, the use of this technique can be very tricky and can result in rather complex relationships. However, this method is at times used. A recent example of the use of this technique to transform from geographic to geomagnetic coordinates can be found in *Mead* (1970).

Another technique is to find the required Euler rotation angles and construct the associated rotation matrices. Then these rotation matrices can be multiplied to give a single transformation matrix (*Goldstein*, 1950). The vector-matrix formalism is attractive not only because it permits a shorthand representation of the transformation, but also because it permits multiple transformations to be performed readily by matrix multiplication and the inverse transformation to be derived readily.

The matrices required for coordinate transformations need not be derived from Euler rotation angles, however. It is the purpose of this note to explain how to derive these coordinate transformations without deriving the required Euler rotation angles as well as to describe the most common coordinate systems in use in the field of solar terrestrial relationships.

Discussions of the coordinate transformations for some of the coordinate systems to be treated in this report may also be found in papers by *Olson* (1970), and by the *Magnetic and Electric Fields Branch* (1970) of the Goddard Space Flight Center. The former paper differs from the present work primarily in notation and the number of systems treated. Another difference is that the Earth's orbit is considered to be circular in Olson's treatment. The latter paper describes coordinate systems and presents the required transformation matrices but without derivation. Since the same coordinate system may receive a different name in each treatment, Table I lists the names and abbreviations used in these two papers and the present work for those systems common to two or more of them.

## TABLE I

Names and abbreviations given to the coordinate systems common to this paper, to the report of *Olson* (1970), and to the report of the *Magnetic and Electric Field Branch* (1970).

Present Work		Olson (1970)		MEFB(1970)	
Coordinate System	Abbr.	Coordinate System	Abbr.	Coordinate System	Abbr.
Geocentric Equatorial Inertial	GEI	-	-	Geocentric Celestial Inertial	GCI
Geographic	GEO	Geographic	G	Geographic	GD
Geomagnetic	MAG	-	-	Geomagnetic	GM
Geocentric Solar Ecliptic	GSE	Ecliptic	EC	Solar Ecliptic	SE
Geocentric Solar Magnetospheric	GSM	Solar Magnetospheric	SM	Solar Magnetospheric	SM
Solar Magnetic	SM	Solar Magnetic	MG	Solar Geomagnetic	SGM

## 2. General Remarks

In defining a coordinate system, in general, you choose two quantities: the direction of one of the axes and the orientation of the other two axes in the plane perpendicular to this direction. This latter orientation is often specified by requiring one of the two remaining axes to be perpendicular to some direction. A fortunate feature of rotation matrices (the matrix that transforms a vector from one system to another) is that the inverse is simply its transpose. Thus, if the matrix  $A$  transforms the vector  $V^a$  measured in system  $a$  to  $V^b$  measured in system  $b$ , then the matrix that transforms  $V^b$  into  $V^a$  is  $A^t$ . Thus we may write

$$\begin{aligned} A \cdot V^a &= V^b \\ A^t \cdot V^b &= V^a \end{aligned}$$

The simplest way to obtain the transformation matrix  $A$  is to find the directions of the three new coordinate axes for system  $b$  in the old system (system  $a$ ). If the direction cosines of the new  $X$ -direction expressed in the old system are  $(X_1, X_2, X_3)$ , of the new  $Y$ -direction are  $(Y_1, Y_2, Y_3)$  and the new  $Z$ -direction are  $(Z_1, Z_2, Z_3)$ , then the rotation matrix is formed by these three vectors as rows, i.e.

$$\begin{aligned} (X_1 \ X_2 \ X_3) (V^{a_x}) &= (V^{b_x}) \\ (Y_1 \ Y_2 \ Y_3) (V^{a_y}) &= (V^{b_y}) \\ (Z_1 \ Z_2 \ Z_3) (V^{a_z}) &= (V^{b_z}) \end{aligned}$$

Similarly the transformation from system  $b$  to  $a$  is

$$\begin{aligned} (X_1 \ Y_1 \ Z_1) (V^{b_x}) &= (V^{a_x}) \\ (X_2 \ Y_2 \ Z_2) (V^{b_y}) &= (V^{a_y}) \\ (X_3 \ Y_3 \ Z_3) (V^{b_z}) &= (V^{a_z}) \end{aligned}$$

The following properties of rotation matrices are useful for error checking. (1) Each row and column is a unit vector. (2) The dot products of any two rows or any two columns is zero. (3) The cross product of any two rows or columns equals the third row or column or its negative. (Row 1 cross row 2 equals row 3; row 2 cross row 1 equals minus row 3.)

### 3. Coordinate Systems

#### 3.1. THE GEOCENTRIC EQUATORIAL INERTIAL SYSTEM

##### 3.1.1. Definition

The Geocentric Equatorial Inertial System (GEI) has its  $X$ -axis pointing from the Earth towards the first point of Aries (the position of the Sun at the vernal equinox). This direction is the intersection of the Earth's equatorial plane and the ecliptic plane and thus the  $X$ -axis lies in both planes. The  $Z$ -axis is parallel to the rotation axis of the Earth and  $Y$  completes the right-handed orthogonal set ( $Y = Z \times X$ ).

##### 3.1.2. Uses

This is the system commonly used in astronomy and satellite orbit calculations. The angles right ascension and declination are measured in this system. If  $(V_x, V_y, V_z)$  is a vector in GEI with magnitude  $V$ , then its right ascension,  $\alpha$ , is  $\tan^{-1}(V_y / V_x)$ ,  $0^\circ \leq \alpha \leq 180^\circ$  if  $V_y \geq 0$ ,  $180^\circ \leq \alpha \leq 360^\circ$  if  $V_y \leq 0$ . Its declination,  $\theta$ , is  $\sin^{-1} V_z / V$ ,  $-90^\circ \leq \theta \leq 90^\circ$ .

### 3.2.

##### 3.2.1. Definition

The geographic coordinate system (GEO) is defined so that its  $X$ -axis is in the Earth's equatorial plane but is fixed with the rotation of the Earth so that it passes through the Greenwich meridian ( $0^\circ$  longitude). Its  $Z$ -axis is parallel to the rotation axis

of the Earth, and its  $Y$ -axis completes a right-handed orthogonal set ( $Y = Z \times X$ ).

### 3.2.2. Uses

This system is used for defining the positions of ground observatories and transmitting and receiving stations. Longitude and latitude in this system are defined in the same way as right ascension and declination in GEI.

### 3.2.3. Transformations

Since the GEO and GEI coordinate systems have their  $Z$ -axes in common, we need only to know the position of the first point in Aries (the  $X$ -axis of GEI) relative to the Greenwich meridian to determine the required transformation. If we let the angle between the Greenwich meridian and the first point of Aries measured eastwards from the first point of Aries in the Earth's equator be  $\theta$ , then the first point of Aries is at  $(\cos\theta, -\sin\theta, 0)$  in the geographic system and the transformation from geographic to GEI is

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEO}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEI}}$$

and the inverse transformation is

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEO}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEI}}$$

The angle  $\theta$  is, of course, a function of the time of day and the time of year, since the Earth spins 366.25 times per year around its axis in inertial space, rather than 365.25 times. Thus, the duration of a day, relative to inertial space, (a sidereal day) is less than 24 h. The angle  $\theta$  is called Greenwich Mean Sidereal Time, and can be calculated by means of the formulas given in [Appendix 2](#).

## 3.3. GEOMAGNETIC COORDINATES

### 3.3.1. Definition

The geomagnetic coordinate system (MAG) is defined so that its  $Z$ -axis is parallel to the magnetic dipole axis. The geographic coordinates of the dipole axis from the International Geomagnetic Reference Field 1965.0 (IGRF) are  $11.435^\circ$  colatitude and  $69.761^\circ$  east longitude (Mead, 1970). Thus the  $Z$ -axis is  $(0.06859, -0.18602, 0.98015)$  in geographic coordinates. The  $Y$ -axis of this system is perpendicular to the geographic poles such that if  $D$  is the dipole position and  $S$  is the south pole  $Y = D \times S$ . Finally, the  $X$ -axis completes a right-handed orthogonal set.

### 3.3.2. Uses

This system is often used for defining the position of magnetic observatories. Also it is a convenient system in which to do field line tracing when current systems, in addition to the Earth's internal field, are being considered (Mead, 1970). The magnetic longitude is measured eastwards from the  $X$ -axis and magnetic latitude is measured from the equator in magnetic meridians, positive northward and negative southwards. Thus, if  $(V_x, V_y, V_z)$  is a vector in the MAG system with magnitude  $V$  then its magnetic longitude,  $\lambda$ , is  $\tan^{-1}(V_y/V_x)$ ,  $0^\circ \leq \lambda < 180^\circ$  if  $V_y \geq 0$ ,  $180^\circ \leq \lambda < 360^\circ$  if  $V_y < 0$ . Its magnetic latitude,  $\theta$ , is  $\sin^{-1} V_z/V$ ,  $-90^\circ \leq \theta \leq 90^\circ$ .

Except near the poles, magnetic longitude will generally be about  $70^\circ$  greater than geographic longitude. We note that a

simple cartesian representation of the dipole magnetic field exists in this system (see [Appendix 1](#)).

### 3.3.3. Transformations

This system is fixed in the rotating Earth and thus the transformation from the geographic coordinate system to the geomagnetic system is constant. From the definitions above we obtain

$$\begin{aligned} (0.33907, -0.91964, -0.19826) (V_x) &= (V_x) \\ (0.93826, 0.34594, 0) (V_y) &= (V_y) \\ (0.06859, 0.18602, 0.98015) (V_z)_{\text{GEO}} &= (V_z)_{\text{MAG}} \end{aligned}$$

## 3.4. GEOCENTRIC SOLAR ECLIPTIC SYSTEM

### 3.4.1. Definition

The geocentric solar ecliptic system (GSE) has its  $X$ -axis pointing from the Earth towards the Sun and its  $Y$ -axis is chosen to be in the ecliptic plane pointing towards dusk (thus opposing planetary motion). Its  $Z$ -axis is parallel to the ecliptic pole. Relative to an inertial system this system has a yearly rotation.

### 3.4.2. Uses

This system has been used to display satellite trajectories, interplanetary magnetic field observations, and solar wind velocity data. The system is useful for the latter display since the aberration of the solar wind can easily be removed in this system because the velocity of the Earth is approximately 30 km/s in the minus  $Y$  direction. However, since the only important effect of the Earth's orbital motion in solar terrestrial relationships is to cause the aberration, other choices of the orientation of the  $Y$  and  $Z$ -axes about the  $X$ -axis have been used. These will be discussed later.

Longitude, as with the geographic system, is measured in the  $X$ - $Y$  plane from the  $X$ -axis toward the  $Y$ -axis and latitude is the angle out of the  $X$ - $Y$  plane, positive for positive  $Z$  components.

### 3.4.3. Transformations

The most common required transformation into the GSE system of those discussed so far is from the GEI system. The direction of the ecliptic pole  $(0, -0.398, 0.917)$  is constant in the GEI system. The  $X$ -axis, the direction of the Sun, may be obtained in GEI from the equations in [Appendix 2](#). If this direction is  $(S_1, S_2, S_3)$ , then the  $Y$ -axis in GEI  $(Y_1, Y_2, Y_3)$  is

$$(0, -0.398, 0.917) \times (S_1, S_2, S_3)$$

and the transformation is

$$\begin{aligned} (S_1 \quad S_2 \quad S_3) (V_x) &= (V_x) \\ (Y_1 \quad Y_2 \quad Y_3) (V_y) &= (V_y) \\ (0 \quad -0.398 \quad 0.917) (V_z)_{\text{GEI}} &= (V_z)_{\text{GSE}} \end{aligned}$$

## 3.5. GEOCENTRIC SOLAR EQUATORIAL SYSTEM

### 3.5.1. Definition

The geocentric solar equatorial system (GSEQ) as with the GSE system has its  $X$ -axis pointing towards the Sun from the

Earth. However, instead of having its  $Y$ -axis in the ecliptic plane, the GSEQ  $Y$ -axis is parallel to the Sun's equatorial plane which is inclined to the ecliptic. We note that since the  $X$ -axis is in the ecliptic plane and therefore is not necessarily in the Sun's equatorial plane, the  $Z$ -axis of this system will not necessarily be parallel to the Sun's axis of rotation. However, the Sun's axis of rotation must lie in the  $X$ - $Z$  plane. The  $Z$ -axis is chosen to be in the same sense as the ecliptic pole, i.e. northwards.

### 3.5.2. Uses

This system has been used extensively to display interplanetary magnetic field data by the Ames magnetometer group (Colburn, 1969). We note that this system is useful for ordering data controlled by the Sun and therefore is an improvement over the use of the GSE system for studying the interplanetary magnetic field and the solar wind. However, for studying the interaction of the interplanetary medium with the Earth yet a third system is more relevant.

### 3.5.3. Transformations

The rotation axis of the Sun,  $R$ , has a right ascension of  $-74.0^\circ$  and a declination of  $63.8^\circ$ . Thus  $R$  is  $(0.122, -0.424, 0.899)$  in GEI. To transform from GEI to GSEQ, we must know the position of the Sun ( $S_1, S_2, S_3$ ) in GEI (see [Appendix 2](#)). Then the  $Y$ -axis in GEI ( $Y_1, Y_2, Y_3$ ) is parallel to  $R \times S$ . Note that since the cross product of two unit vectors is not a unit vector unless they are perpendicular to each other, this cross product must be normalized. Finally the  $Z$ -axis in GEI ( $Z_1, Z_2, Z_3$ ) =  $S \times Y$ . Then

$$\begin{pmatrix} S_1 & S_2 & S_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEI}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GSEQ}}$$

Since both GSE and GSEQ coordinate systems have their  $X$ -axes directed towards the Sun, they differ only by a rotation about the  $X$ -axis. Thus the transformation matrix from GSE to GSEQ must be of the form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GSE}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GSEQ}}$$

If the transformations from GEI to GSE and GEI to GSEQ are both known, then the angle may be determined by examining the angle between the  $Y$ -axes in the two systems or the  $Z$ -axes (i.e. the angle between the vectors formed by the second row of each matrix or the third row). If these transformation matrices are not available, may be calculated from the following formula

$$\sin \theta = S \cdot (0.031, -0.112, -0.049) / |(0.122, -0.424, 0.899)| \times S$$

where  $S$  is the position of the Sun in GEI and can be calculated from the formulas in [Appendix 2](#). Since the Sun's spin axis is inclined  $7.25^\circ$  to the ecliptic, ranges from  $-7.25^\circ$  (on approximately Dec. 5) to  $7.25^\circ$  (on June 5) each year. The Sun's spin axis is directed most towards the Earth on approximately Sept. 5 at which time the Earth reaches its most northerly heliographic latitude. At this time  $\theta$  equals 0.

## 3.6. GEOCENTRIC SOLAR MAGNETOSPHERIC SYSTEM

### 3.6.1. Definition

The geocentric solar magnetospheric system (GSM), as with both the GSE and GSEQ systems, has its  $X$ -axis from the Earth to the Sun. The  $Y$ -axis is defined to be perpendicular to the Earth's magnetic dipole so that the  $X$ - $Z$  plane contains the dipole axis. The positive  $Z$ -axis is chosen to be in the same sense as the northern magnetic pole. The difference between the

GSM system and the GSE and GSEQ is simply a rotation about the  $X$ -axis.

### 3.6.2. Uses

This system is useful for displaying magnetopause and shock boundary positions, magnetosheath and magnetotail magnetic fields and magnetosheath solar wind velocities because the orientation of the magnetic dipole axis alters the otherwise cylindrical symmetry of the solar wind flow. It also is used in models of magnetopause currents (*Olson, 1969*). It reduces the three dimensional motion of the Earth's dipole in GEI, GSE, etc., to motion in a plane (the  $X$ - $Z$  plane). The angle of the north magnetic pole to the GSM  $Z$ -axis is called the dipole tilt angle and is positive when the north magnetic pole is tilted towards the Sun. In addition to a yearly period due to the motion of the Earth about the Sun, this coordinate system rocks about the solar direction with a 24 h period. We note that since the  $Y$ -axis is perpendicular to the dipole axis, the  $Y$ -axis is always in the magnetic equator and since it is perpendicular to the Earth-Sun-line, it is in the dawn-dusk meridian (pointing towards dusk). GSM longitude is measured in the  $X$ - $Y$  plane from  $X$  towards  $Y$  and latitude is the angle northward from the  $X$ - $Y$  plane. However, another set of spherical polar angles is sometimes used. Here the angle, between the vector and the  $X$ -axis, called the Sun-Earth probe angle (SEP) or the Sun-Earth-satellite angle (SES) is the polar angle and the angle of the projected vector in the  $Y$ - $Z$  plane is the azimuthal angle. It is measured from the positive  $Y$ -axis towards the positive  $Z$ -axis.

### 3.6.3. Transformations

To transform from GEI to GSM we need to know both the position of the Sun in GEI and the position of the Earth's dipole axis. The position of the Sun  $S$  ( $S_1, S_2, S_3$ ) can be obtained from [Appendix 2](#). The position of the dipole  $D$  must be obtained by transforming from geographic coordinates (see [Section 2](#)). In geographic coordinates, the dipole is at  $11.435^\circ$  colatitude and  $69.761^\circ$  east longitude (IGRF epoch 1965.0). Thus,  $D$  in geographic coordinates is  $(0.06859 -0.18602, 0.98015)$ . If  $D'$  is  $D$  transformed into GEI, the  $Y$ -axis is

$$D' \times S / |D' \times S|$$

We note that the normalizing factor occurs because  $D'$  and  $S$  are not necessarily perpendicular. Finally,  $Z$  is  $S \times Y$  and the transformation becomes

$$\begin{pmatrix} S_1 & S_2 & S_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GSE}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GSM}}$$

The transformation matrix between GSM and GSE or GSEQ is of the form

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

However, since changes both with time of day and time of year, it is not derivable from a simple equation. However, if the transformation matrix from GEI to GSE,  $A_{\text{GSE}}$  and from GEI to GSM,  $A_{\text{GSM}}$  are both known, then the transformation from GSM to GSE is simple  $A_{\text{GSE}} A_{\text{GSM}}^{-1}$  where  $A_{\text{GSE}}^{-1}$  is the transpose of  $A_{\text{GSE}}$ . An analogous formula holds for the transformation from GSM to GSEQ. We note that the amplitude of the diurnal variation of is  $11.4^\circ$  which is added to an annual variation of  $23.5^\circ$ .

## 3.7. SOLAR MAGNETIC COORDINATES

### 3.7.1. Definition

In solar magnetic coordinates (SM) the Z-axis is chosen parallel to the north magnetic pole and the Y-axis perpendicular to the Earth-Sun line towards dusk. The difference between this system and the GSM system is a rotation about the Y-axis. The amount of rotation is simply the dipole tilt angle as defined in the previous section. We note that in this system the X-axis does not point directly at the Sun. As with the GSM system, the SM system rotates with both a yearly and daily period with respect to inertial coordinates.

### 3.7.2. Uses

The solar magnetic system is useful for ordering data controlled more strongly by the Earth's dipole field than by the solar wind. It has been used for magnetopause cross sections and magnetospheric magnetic fields. We note that since the dipole axis and the Z-axis of this system are parallel the cartesian components of the dipole magnetic field are particularly simple in this system (see [Appendix 1](#)).

### 3.7.3. Transformations

As for GSM, the transformation from GEI to SM requires a knowledge of the Earth Sun direction  $S$ , and the dipole direction  $D$  in GEI. Having obtained these as in [Section 3.6](#), we find  $Y=(D \times S)/(D \times S)$  and  $X=Y \times D$ . Then the transformation becomes

$$\begin{pmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ D_1 & D_2 & D_3 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GEI}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{SM}}$$

The transformation from GSM to SM is simply a rotation about the Y-axis by the dipole tilt angle  $\mu$ . Thus

$$\begin{pmatrix} \cos\mu & 0 & -\sin\mu \\ 0 & 1 & 0 \\ \sin\mu & 0 & \cos\mu \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{GSM}} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_{\text{SM}}$$

## 3.8. DIPOLE MERIDIAN SYSTEM

### 3.8.1. Definition

As with the solar magnetic system, the Z-axis of the dipole meridian system (DM) is chosen along the north magnetic dipole axis. However, the Y-axis is chosen to be perpendicular to a radius vector to the point of observation rather than the Sun. The positive Y direction is chosen to be eastwards, so that the X-axis is directed outwards from the dipole. This is a local coordinate system, in that it varies with position, however, since the X-Z plane contains the dipole magnetic field it is quite useful.

### 3.8.2. Uses

It is used to order data controlled by the dipole magnetic field where the influence of the solar wind interaction with the magnetosphere is weak. It has been used extensively to describe the distortions of the magnetospheric field in terms of the two angles declination and inclination which can be easily derived from measurements in this system (*Mead and Cahill, 1967*). The inclination,  $I$ , is simply the angle that the field makes with the radius vector minus 90. Thus, if  $\mathbf{R}$  is the unit vector from the center of the Earth to the point of observation in the DM system (we note that in this system  $R_y=0$ ), and  $\mathbf{b}$  is the direction of the magnetic field in the DM system, then  $I = \cos^{-1}(\mathbf{R} \cdot \mathbf{b}_x + \mathbf{R}_z \cdot \mathbf{b}_z) - 90^\circ$ . The declination,  $D$ , is measured about the radius vector with  $D=0$  in the X-Z plane and positive  $D$  angles for positive  $b_x$ . Thus  $D = \tan^{-1}[b_y/(\mathbf{R} \cdot \mathbf{b}_z + \mathbf{R}_z \cdot \mathbf{b}_x)]$ ,  $0^\circ \leq D \leq 180^\circ$  for  $0 \leq b_y \leq 1$  and  $0^\circ \geq D \geq 180^\circ$  for  $0 \geq b_y \geq -1$ . As in the SM system, the cartesian components of the dipole field can be expressed very simply in this system. In particular,  $B_y = 0$  by definition.

### 3.8.3. Transformations

To transform from any system to the dipole meridian system we must know the dipole axis,  $\mathbf{D}$ , in this system, and the unit position vector of the point of observation relative to the center of the Earth. Since  $\mathbf{Y}$  is perpendicular to  $\mathbf{R}$  and  $\mathbf{D}$  then  $\mathbf{Y} = (\mathbf{D} \times \mathbf{R}) / (D \times R)$  and  $\mathbf{X} = \mathbf{D} \times \mathbf{Y}$ . Thus

$$\begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix} (\mathbf{V}_x) = (\mathbf{V}_x) \\ \begin{pmatrix} Y_1 & Y_2 & Y_3 \end{pmatrix} (\mathbf{V}_y) = (\mathbf{V}_y) \\ \begin{pmatrix} D_1 & D_2 & D_3 \end{pmatrix} (\mathbf{V}_z) = (\mathbf{V}_z)_D$$

We note that this transformation usually is particularly straight forward from geographic coordinates because the geographic latitude and longitude of a point of observation is often known and the dipole is fixed in geographic coordinates. From geomagnetic coordinates it is simple rotation about the  $Z$ -axis by the magnetic longitude. From solar magnetic co-ordinates, it is a rotation about the  $Z$ -axis by the angle between the projections of the Sun and the local radius vector in the magnetic equator.

## 3.9. ATS-1 COORDINATE SYSTEMS

Two coordinate systems have been used extensively in the analysis of the magnetometer data from the ATS-1 satellite which differ slightly from previously described coordinate systems. The ATS  $XYZ$  system is the coordinate system in which the ATS magnetometer data are originally obtained. The  $Z$ -axis is parallel to the Earth's rotation axis. Thus, it is parallel to the  $Z$ -axis of the geographic, and GEI systems. However, the  $Y$ -axis is chosen perpendicular to the Earth-Sun line towards dusk. The  $X$ -axis completes a right handed orthogonal set. Thus the  $X$ - $Y$  plane is the Earth's rotational equator with  $X$  in the noon meridian.

The other ATS coordinate system is ATS  $VDH$ . In this system  $H$  is chosen parallel to the Earth's spin axis.  $V$  is the local vertical. Since ATS-1 is in the Earth's equatorial plane,  $V$  is perpendicular to  $H$ . Finally,  $D$ , completes the right-handed set ( $\mathbf{D} = \mathbf{H} \times \mathbf{V}$ ) and is azimuthal, eastwards in the equatorial plane. The transformation between the ATS  $XYZ$  and ATS  $VDH$  systems is

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} (\mathbf{B}_x) = (\mathbf{B}_v) \\ \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} (\mathbf{B}_y) = (\mathbf{B}_D) \\ \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} (\mathbf{B}_z) = (\mathbf{B}_H)$$

where  $\theta$  (deg) = 15 (UT + 2 h).

## 3.10 OTHER COORDINATE SYSTEMS

All the coordinate systems described so far have been geocentric and, with the exception of the dipole meridian system and the ATS  $VDH$  system, have been independent of the position of the point of observation. When considering measurements far from the Earth, it is often useful to choose coordinate systems which are dependent on the position of the observation point rather than the position of the Earth. For example, *Coleman et al.* (1969) use a system analogous to the GSEQ system but with the Mariner 4 - Sun line as the  $X$ -axis. We note, however, they have chosen their three axes anti-parallel to the axes of the analogous GSEQ system and thus their right-handed triad of coordinates is a noncyclic permutation of these three antiparallel vectors. For studying solar-planetary interactions, the required modifications to alter the transformations given in the previous sections to those relevant to the problem being considered should be obvious.

However, there is another class of cartesian coordinate systems that can be used: those based on a local measurement. For example, one may wish to define a coordinate system in which the solar wind flow is parallel to one of the coordinate axes.

This could be done in coordinate systems such as GSE, GSEQ and GSM by replacing the position of the Sun by the vector antiparallel to the observed solar wind flow. The second condition for choosing the coordinate system would be that the  $Y$ -axis is perpendicular to the solar wind and the ecliptic pole (for GSE) and the Sun's rotation axis (for GSEQ) and the Earth's dipole (for GSM). However, we note that in GSE, the  $Z$ -axis will no longer necessarily be parallel to the ecliptic pole since the solar wind flow need not be in the ecliptic plane.

Another way of choosing the system is to choose one axis along the measured magnetic field. As before, we are now left with the choice of the orientation of the other two axes about this one. In the solar wind it is often useful to choose one of these two axes perpendicular to the plane defined by the magnetic field and the solar wind flow velocity. In the magnetosphere, it is convenient to choose one of these two axes to be perpendicular to a dipole magnetic meridian.

Finally, since it is much easier to visualize data and spacecraft trajectories in two dimensions rather than three, mention should be made of a two dimensional coordinate system in common use. Since the solar wind, neglecting the magnetic field is approximately cylindrically symmetric about the radial direction from the Sun, if it interacts with a figure of revolution about the Earth-Sun line such as a planet, the interaction should be the same in every plane containing the planet-Sun line. In other words, while the interaction may be a function of radial distance and the angle away from the planet-Sun line (SES or SEP angle in the case of the Earth), it is not a function of the azimuthal angle around the planet Sun line. The Earth's magnetosphere is not cylindrically symmetric about the solar wind flow. However, in the dawn-dusk plane the calculated magnetopause position should deviate less than about 20% from cylindrical (*Olson, 1969*). Thus, it is not unreasonable at times to assume cylindrical symmetry for the interaction.

This coordinate system may be thought of in several ways. (1) It is a cylindrical coordinate system with the variables  $r, \theta, X$  where  $r$  is the distance from the axis of the cylinder,  $X$  is the distance along the axis, and  $\theta$  is the angle around the axis. In plotting a spacecraft trajectory in this system, we would plot  $r$  vs  $X$ . (2) It is a polar coordinate system where we plot the magnitude of the vector versus the angle between the vector and the planet-Sun line. (3) It is a two dimensional cartesian coordinate system where we plot the component along the planet-Sun line versus the square root of the sum of the squares of the other two components. This system has been used to describe the trajectory of spacecraft near encounters with other planets and to plot the positions of magnetopause and bow shock crossings by Earth orbiting spacecraft.

## Appendix 1. The Cartesian Representation of a Dipole Magnetic Field

The usual representation of a dipole magnetic field is one which separates the field into a radial and tangential component. This gives the magnetic field in a local two dimensional coordinate system. However, a very simple representation of the field exists in a cartesian coordinate system also (*Alfven and Falthammar, 1963*). If  $(X, Y, Z)$  is the location of the point of observation in solar magnetic coordinates, the field due to the Earth's dipole is

$$B_x = 3XZ (B_0/R^5)$$

$$B_y = 3YZ (B_0/R^5)$$

$$B_z = (3Z^2 - R^2) (B_0/R^5)$$

where  $R^2 = X^2 + Y^2 + Z^2$  and  $B_0$  is the magnetic moment of the Earth.  $B_0$  is numerically equal to the field at the equator on the surface of the Earth if distances are measured in Earth radii.

We note that the same formula is valid for any coordinate system which is a rotation about the dipole from the solar magnetic coordinate system. In particular, it is valid for the dipole meridian system in which case  $B_y = 0$ . With the knowledge of the dipole tilt angle the above representation also allows a simple derivation of the dipole field in GSM coordinates (cf. Section 7).

## Appendix 2. The Calculation of the Position of the Sun

G.D. Mead (private communication) has written a simple subroutine to calculate the position of the Sun in GEI coordinates. It is accurate for years 1901 through 2099, to within 0.006 deg. The input is the year, day of year and seconds of the day in UT. The output is Greenwich Mean Sideral Time in degrees, the ecliptic longitude, apparent right ascension and declination of the Sun in degrees. The listing of this program follows. We note that the cartesian coordinates of the vector from the Earth to the Sun are:

$$\begin{aligned} X &= \cos(\text{SRASN}) \cos(\text{SDEC}) \\ Y &= \sin(\text{SRASN}) \cos(\text{SDEC}) \\ Z &= \sin(\text{SDEC}) \end{aligned}$$

```

SUBROUTINE SUN(IYR, IDAY, SECS, GST, SLONG, SRASN, SDEC)
C PROGRAM TO CALCULATE SIDEREAL TIME AND POSITION OF THE SUN.
C GOOD FOR YEARS 1901 THROUGH 2099. ACCURACY 0.006 DEGREE.
C INPUT IS IYR, IDAY (INTEGERS), AND SECS, DEFINING UN. TIME.
C OUTPUT IS GREENWICH MEAN SIDEREAL TIME (GST) IN DEGREES,
C LONGITUDE ALONG ECLIPTIC (SLONG), AND APPARENT RIGHT ASCENSION
C AND DECLINATION (SRASN, SDEC) OF THE SUN, ALL IN DEGREES
DATA RAD /57.29578/
DOUBLE PRECISION DJ, FDAY
IF(IYR. LT. 1901. OR. IYR. GT. 2099) RETURN
FDAY = SECS/86400
DJ = 365* (IYR-1900) + (IYR-1901)/4 + IDAY + FDAY -0.5D0
T = DJ / 36525
VL = DMOD (279.696678 + 0.9856473354*DJ, 360.D0)
GST = DMOD (279.690983 + 0.9856473354*DJ + 360.*FDAY + 180., 360.D0)
G = DMOD (358.475845 + 0.985600267*DJ, 360.D0) / RAD
SLONG = VL + (1.91946 -0.004789*T)*SIN(G) + 0.020094*SIN (2.*G)
OBLIQ = (23.45229 -0.0130125*T) / RAD
SLP = (SLONG -0.005686) / RAD
SIND = SIN (OBLIQ)*SIN (SLP)
COSD = SQRT(1.-SIND**2)
SDEC = RAD * ATAN (SIND/COSD)
SRASN = 180. -RAD*ATAN2
(COTAN (OBLIQ)*SIND/COSD, -COS (SLP)/COSD)
RETURN
END

```

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